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The Sensitivity Analysis of Yaw Rate for a Front Wheel Steering Vehicle: In the Frequency Domain

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In this paper, the sensitivity analysis of yaw rate for a front wheel steering vehicle is performed in the frequency domain. A two degree of freedom system is used to represent a simple vehicle model in order to derive the system transfer function. The transfer function is defined between a steering angle input and a yaw rate output. This model shows the simplest lateral dynamic effect and is useful for understanding of the dynamic characteristics and control aspect of the target system. Vehicle mass, inertia, cornering stiffness, and wheel base are selected as design variables. Sensitivity functions of the transfer function with repect to design variables are derived, and the results show that the response of the yaw rate always has stable minimum phase characteristics. The objective of this paper is the proposition of a base for re-design and new-design of the vehicle by checking the yaw rate variations with respect to the design variable change in the frequency domain. Finally, dominant design variables can be selected based on the sensitivity analysis.

Key Words: Sensitivity Analysis, Front Wheel Steering Vehicle, Frequency Domain, Lateral Vehicle Dynamics, Yaw Rate, Steering Characteristics, Minimum Phase

Nomenclature

- m : Vehicle mass
- I : Moment of intrtia of a vehicle in yaw direction
- V : Vehicle speed
- K_f : Cornering stiffness of a front wheel
- K_r : Cornering stiffness of a rear wheel
- l_f : Distance from c.g. to front wheel center
- l_r : Distance from c.g. to rear wheel center
- δ_f : Front steering angle

1. Introduction

The sensitivity analysis is an efficient tool for checking the effects of the design variables on the system state variables (Deif, 1986). State variables represent dynamic characteristics of the system. Otherwise, design variables are specified from the design criteria. So, characteristics of state variables are governed by the initial values of design variables. In case of re-designing the sysyem, sensitivity information could be used as a design base (Vanderplaats, 1984; Arora, 1989). Therefore, sensitivity analysis might be used as an important tool for the designer.

The sensitivity analysis with respect to system states can be classified by static, kinematic, and dynamic ones. The static sensitivity analysis is used to check the variations of the state variables with respect to design variable changes in static state. Particularly, this could be utilized in the structural design. The kinematic sensitivity analysis could be applied in kinematic state of the mechanism (Haug and Sohoni, 1984). The dynamic sensitivity analysis is utilized for evaluation of the variation of the target mechanism in the dynamic stat (Krishnaswami et al., 1983, Haug et al., 1984, Chang and Nikravesh, 1985, Jang and Han, 1995). This sensitivity analysis is usually performed in the time domain. For evalu-

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ation of the steady state characteristics of the target system, the frequency domain analysis is necessary. A sensitivity analysis of the side slip angle of a front wheel steering vehicle in the frequency domain was performed by Jang and Han, 1996. From this research, they show the reponse of the side slip angle has two distinct sub-domains (minimum and nonminimum phase). All useful sensitivity formulations and results in the frequency domain have been suggested by the form of the equations and tables.

In this paper, the sensitivity analysis of the yaw rate of the front wheel steering vehicle is considered in the frequency domain. The lateral vehicle dynamics are concerned in steering maneuver when an aoutmobile is driven with variable forward speed. Generally, the steering maneuver of the vehicle is carried out by the steering wheel input of the driver. Then the vehicle is steered to the target path of the driver's desire. This is one of the important maneuvers for the maneuverability of the vehicle (Whitehead, 1988). Especially the front wheel steering vehicle is commonly used in most passenger cars.

In order to study the sensitivity analysis of yaw rate and checking the validity of the developed control logic, a simple bicycle model is widely used. It could be also used for evaluation of the base lateral dynamic response (Ellis, 1969; Gillespic, 1992; Shiotsuka, Nagamstsu, and Yoshida, 1993). This model is the simplest one for the verification of some type of steering maneuver (Jansen and van Oosten, 1994).

In the steering maneuvers of the front wheel steering vehicle, two dominant state variables, the side slip angle and yaw ratre, are conventionally used. In this paperl, the yaw rate reponse in the frequency domain is studied. For this purpose, a transfer function and some useful basic terms (SF, ξ, and ω_n) are defined and used to understand the steady state characteristics of the vehicle. These terms are also used for formulation of the sensitivity functions. Magnitude and phase response are evaluated, and sensitivity results of the response are suggested by these derived sensitivity functions. All formulations for the sensitivity analysis are performed by the first order partial differentiation with repect to the design variables.

The objective of this research is a proposition of the sensitivity information for the magnitude and phase of the yaw rate with respect to the design variable change. Finally, for the re-design and new-design of the vehicle, dominant design variables of the sysyem are suggested based on the analyses in the frequency domain.

2. Vehicle Modeling

Although various vehicle models that could be used for this purpose, a simple bicycle vehicle model is selected for the analytical derivation of the sensitivity equations with respect to some design variables. A vehicle in steering is represented in Fig. 1 and the major concern is about the lateral vehicle dynamics in steering maneuver.

Each parameter and design variable is listed in nomenclature. The side slip angle and yaw rate are selected as the state variables of the system,

$$\underline{z}=[\beta \ r]^T \tag{1}$$

where $\underline{z}$ means the state variable vector, β is side slip angle and r represents yaw rate of the vehicle. Upper script T means transpose of a vector or matrix. Design variables of this system are selected as the following forms:

$$\begin{aligned}\underline{b}&=[b_1 \ b_2 \ b_3 \ b_4 \ b_5 \ b_6]^T\\&=[m \ I \ K_f \ K_r \ l_f \ l_r]^T\end{aligned} \tag{2}$$

A mathematical governing equation of motion for the front wheel steering vehicle can be expres-

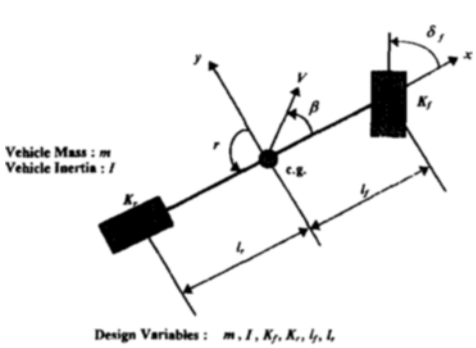

Fig. 1 A vehicle in steering maneuver

sed as Eqs. (3) and (4) (Jang, 1995). These equations could be derived with assumptions that side slip angle is very small and K_f and K_r have linear linear characteristics in normal driving conditions. Also, weight transfer between the right and the left of the vehicle is ignored.

$$mV\frac{d\beta}{dt}+2(K_f+K_r)\beta + \left[mV+\frac{2}{V}(l_fK_f-l_rK_r)\right]r=2K_f\delta_f \quad (3)$$

$$2(l_fK_f-l_rK_r)\beta + I\frac{dr}{dt}+\left[\frac{2(l_f^2K_f+l_r^2K_r)}{V}\right]r=2l_fK_f\delta_f \quad (4)$$

Equations (3) and (4) are the basic equations for two dimensional plane motion of the vehicle. The left sides of the Eqs. (3) and (4) represent motion of the vehicle and the right sides of the equations mean input to the vehicle with steer angle(δ_f). From Eqs. (3) and (4), taking Laplace transformation for each side of the equations, we could obtain the following equations:

$$[mVs+2(K_f+K_r)]\beta(s) + \left[mV+\frac{2}{V}(l_fK_f-l_rK_r)\right]r(s) = 2K_f\delta_f(s) \quad (5)$$

$$2(l_fK_f-l_rK_r)\beta(s) + \left[Is+\frac{2(l_f^2K_f+l_r^2K_r)}{V}\right]r(s) = 2l_fK_f\delta_f(s) \quad (6)$$

where $\beta(s)$, $r(s)$ and $\delta_f(s)$ are Laplace transformation variables of the β, r, and δ_f, respectively. After some manipulation of the Eqs. (5) and (6), the following input/output relations could be derived. In this case, steer angle is an input and yaw rate is an output variable.

$$\frac{r(s)}{\delta_f(s)}=\frac{\begin{vmatrix} mVs+2(K_f+K_r) & 2K_f \\ 2(l_fK_f-l_rK_r) & 2l_fK_f \end{vmatrix}}{\begin{vmatrix} mVs+2(K_f+K_r) & mV+\frac{2}{V}(l_fK_f-l_rK_r) \\ 2(l_fK_f-l_rK_r) & Is+\frac{2}{V}(l_f^2K_f+l_r^2K_r) \end{vmatrix}} \quad (7)$$

Generally, when governing when governing equations of the mechanical systems are given, the yaw reponse for the steer angle can be obtained under the aequate driveng conditions. The characteristic equations of the system could be easily obtained by setting the denominator of the Eq. (7) to zero. Finally, refined form of the characteristic equation for this case is:

$$s^2+2\xi\omega_n+\omega_n^2=0 \quad (8)$$

where,

$$2\xi\omega_n=\frac{2m(l_f^2K_f+l_r^2K_r)+2I(K_f+K_r)}{mIV} \quad (9)$$

$$\omega_n^2=\frac{4K_fK_rl^2}{mIV^2}-\frac{2(l_fK_f-l_rK_r)}{I} \quad (10)$$

If ω_n means a natural frequency and ξ is a damping ratio of the system, final forms of the these terms are obtained like as Eqs. (11) and (12).

$$\omega_n=\frac{21}{V}\sqrt{\frac{K_fK_r}{mI}}\sqrt{1+SF\cdot V^2} \quad (11)$$

$$\xi=\frac{m(l_f^2K_f+l_r^2K_r)+I(K_f+K_r)}{2l\sqrt{mIK_fK_r(1+SF\cdot V^2)}} \quad (12)$$

where, SF is a stability factor of the vehicle and is expressed as follows:

$$SF=-\frac{m}{2l^2}\frac{l_fK_f-l_rK_r}{K_fK_r} \quad (13)$$

This term is known as one of the understeer/oversteer gradients. If a vehicle has a positive value of the SF, it shows an understeer characteristic. Like Eqs. (11) and (12), the natural frequency and damping ratio are functions of the design variable and speed of the vehicle.

Finally, a transfer function between steering angle input and yaw rate output could be expressed with the previous informations. It is a function of the vehicle design variables and some basic vehicle functions (SF, ξ, and ω_n). Since our major concern is about the response of the yaw rate in the requency domain, the following transfer function could be obtained from Eq. (7):

$$G_\delta^r(s)=\frac{r(s)}{\delta_f(s)} = G_\delta^r(0)\frac{1+T_rs}{1+\frac{2\xi}{\omega_n}s+\left(\frac{1}{\omega_n}\right)^2 s^2} \quad (14)$$

where,

$$G_{\delta}^{\gamma}(0)=\frac{1}{1+SF\cdot V^2}\cdot\frac{V}{l} \quad (15)$$

$$T_r=\frac{ml_fV}{2lK_r} \quad (16)$$

where $l(=l_f+l_r)$ represents wheel base of a vehicle and $G_{\delta}^{\gamma}(0)$ is a yaw rate gain coefficient which could be defined as the value of the yaw rate with respect to the steer angle in a steady state circular turning maneuver. And, T_r means a coefficient of the s in the numerator of the transfer function (Eq. (14)).

Since T_r always has a positive quantity, we could note that the yaw response (Eq. (14)) has stable zero dynamics. This means that the response has minimum phase characteristics. (Ogata, 1990, Slotine and Li, 1991, and Franklin et al., 1994). It will be verified through some analyses in the frequency domain.

3. Formulation for the Sensitivity Analysis

In this chapter, sensitivity functions on the yaw rate with respect to the design variables are derived. These are could be classified into sensitivity functions for the basic terms and magnitude and phase of the transfer function for more systematic approach.

3.1 Sensitivity functions for the basic terms

For the formulation process of the sensitivity functions for yaw rate with respect to the design variables, sensitivity formulations for the basic terms(SF, ξ, and ω_n) are required at the initial stage. This information is usful for derivation of the sensitivity of magnitude and phase of the trasfer function.

First, the stability factor for a vehicle could be defined as a function of the vehicle parameters from Eq. (13). It is represented by the following form of the equation:

$$SF=f(\underline{b})$$
$$\frac{\partial SF}{\partial \underline{b}}=\frac{\partial f}{\partial \underline{b}} \quad (17)$$

A sensitivity function for the stabilit factor (Eq. (17)) is the partial derivative of the stability factor with respect to the design variable vector (Jand and Han, 1996). This information will be used for formulation of a natural frequency, damping ratio, magnitude, and phase of the vehicle.

Secondly, by the same reason, the damping ratio for a vehicle could be defined as a function of the vehicle parameters and forward vehicle speed from Eq. (12). It is expressed by the following form of the equation:

$$\xi=g(\underline{b})$$
$$\frac{\partial \xi}{\partial \underline{b}}=\frac{\partial g}{\partial \underline{b}} \quad (18)$$

A sensitivity functions for damping ration (Eq. (18)) is also defined as a partial derivative of the damping ratio with respect to the design variables.

Finally, the last term of the basic functions in this study is the natural frequency for a vehicle. It is defined as a function of vehicle parameters and forward vehicle speed from Eq. (11). It could be represented by the following form of the equation for systematic formulation process:

$$\omega_n=h(\underline{b})$$
$$\frac{\partial \omega_n}{\partial \underline{b}}=\frac{\partial h}{\partial \underline{b}} \quad (19)$$

A sensitivity function for natural frequency (Eq. (19)) is defined as a partial derivative of the natrual frequency with respect to the design variables. This sensitivity information for damping ratio and natural frequency will be used for formulation of the magnitude and phase of the vehicle.

3.2 Sensitivity functions for the magnitude and phase

From the Eq. (14), sensitivity functions for the magnitude and phase of the transfer function could be derived. And, contents of the previous section are used for the systematic derivation process. First, magnitude and phase of the transfer function have to be dizefined as follows. In order to check the frequency response, the Laplace variables have to be replaced by the following terms $s=j2\pi f$. In this case, f means that the forcing input frequency(Hz) to the vehicle and j is an indicator of the imaginary part for the complex variable. Therefore, the magnitude of the transfer function is:

$$|G^r_\delta(j2\pi f)|=A\cdot B \tag{20}$$

where,

$$A=|G^r_\delta(0)|$$

$$B=\sqrt{\frac{P_r^2+Q_r^2}{R_r^2+S_r^2}}$$

And, phase of the transfer function is:

$$\angle G^r_\delta(j2\pi f) = tan^{-1}\left(\frac{Q_r}{P_r}\right)-tan^{-1}\left(\frac{S_r}{R_r}\right) \tag{21}$$

where, P_r, Q_r, R_r, and S_r are defined as the following terms:

$$P_r=1$$

$$Q_r=(2\pi f)T_r$$

$$R_r=1-\left(\frac{2\pi f}{\omega_n}\right)^2$$

$$S_r=2\frac{\xi}{\omega_n}(2\pi f) \tag{22}$$

As the first process of the derivation of the sensitivity functions in the frequency domain, sensitivity of the magnitude must be considered. If we perform the partial differentiation of the magnitude with repect to the design variables, the following general sensitivity function for magnitude can be obtained from Eq. (20):

$$\frac{\partial}{\partial \underline{b}}|G^r_\delta(j2\pi f)|=A_{\underline{b}}\cdot B+A\cdot B_{\underline{b}} \tag{23}$$

where $A_{\underline{b}}$ and $B_{\underline{b}}$ are partal aerivatrives of A and B with respect to the design variable vector $\underline{b}$. For the evaluation of the sensitivity functions for magnitude, let's divide Eq. (23) into small parts. In order to a cquire the sensitivity of A in Eq. (23) with respect to the design variables, the variables, the yaw rate gain coefficient could be re-defined from Eq. (15):

$$A=\frac{g(\underline{b})}{f(\underline{b})} \tag{24}$$

where,

$$g(\underline{b})=\frac{V}{l}$$

$$f(\underline{b})=1+SF\cdot V^2$$

Finally, the sensitivity of A with respect to the design variables is derived. Also, the exct expressions of the sensitivity function for the sub-functions have been summarized in Table 1:

$$A_{\underline{b}}=\frac{\partial}{\partial \underline{b}}\left[\frac{g}{f}\right]=\frac{1}{f^2}\left[\frac{\partial g}{\partial \underline{b}}f-g\frac{\partial f}{\partial \underline{b}}\right] \tag{25}$$

Table 1 Sensitivity Functions for g and f in A

Design Variables \ Functions	g	f
m	0	$\frac{\partial SF}{\partial m}V^2$
I	0	$\frac{\partial SF}{\partial I}V^2$
K_f	0	$\frac{\partial SF}{\partial K_f}V^2$
K_r	0	$\frac{\partial SF}{\partial K_r}V^2$
l_f	$-\frac{V}{(l_f+l_r)^2}$	$\frac{\partial SF}{\partial l_f}V^2$
l_r	$-\frac{V}{(l_f+l_r)^2}$	$\frac{\partial SF}{\partial l_r}V^2$

where the sensitivity functions of the stalility factor with respect to the design variables were obtained by the basic sensitivity results (Jand and Han, 1996).

The sensitivity function of the B in Eq. (23) with respect to the design variables is obtained by the following process:

$$B_{\underline{b}}=\frac{1}{2}\sqrt{\frac{R_r^2+S_r^2}{P_r^2+Q_r^2}}\cdot\frac{\partial}{\partial \underline{b}}\left(\frac{P_r^2+Q_r^2}{R_r^2+S_r^2}\right) \tag{26}$$

where,

$$\frac{\partial}{\partial \underline{b}}\left(\frac{P_r^2+Q_r^2}{R_r^2+S_r^2}\right)=\frac{\left(\left[\frac{\partial}{\partial \underline{b}}(P_r^2+Q_r^2)\right]\left(R_r^2+S_r^2\right)-(P_r^2+Q_r^2)\left[\frac{\partial}{\partial \underline{b}}(R_r^2+S_r^2)\right]\right)}{(R_r^2+S_r^2)^2} \tag{27}$$

also where,

$$\frac{\partial}{\partial \underline{b}}(P_r^2+Q_r^2)=2P_r\frac{\partial P_r}{\partial \underline{b}}+2Q_r\frac{\partial Q_r}{\partial \underline{b}} \quad (28)$$

$$\frac{\partial}{\partial \underline{b}}(R_r^2+S_r^2)=2R_r\frac{\partial R_r}{\partial \underline{b}}+2S_r\frac{\partial S_r}{\partial \underline{b}} \quad (29)$$

where, the partial derivatives of P_r, Q_r, R_r, and S_r with respect to the design variables are defined as follows:

$$\begin{aligned} &\frac{\partial P_r}{\partial \underline{b}}=0 \\ &\frac{\partial Q_r}{\partial \underline{b}}=(2\pi f)\frac{\partial T_r}{\partial \underline{b}} \\ &\frac{\partial R_r}{\partial \underline{b}}=(2\pi f)^2\frac{2}{\omega_n^3}\frac{\partial \omega_n}{\partial \underline{b}} \\ &\frac{\partial S_r}{\partial \underline{b}}=(4\pi f)\frac{\partial}{\partial \underline{b}}\left(\frac{\xi}{\omega_n}\right) \end{aligned} \quad (30)$$

also where,

$$\frac{\partial}{\partial \underline{b}}\left(\frac{\xi}{\omega_n}\right)=\frac{\frac{\partial \xi}{\partial \underline{b}}\omega_n-\frac{\partial \omega_n}{\partial \underline{b}}\xi}{\omega_n^2}$$

The sensitivity functions of T_r could be derived by the same procedures.

$$T_r=\frac{g_{Tr}(\underline{b})}{f_{Tr}(\underline{b})} \quad (31)$$

where,

$$g_{Tr}(\underline{b})=ml_fV$$
$$f_{Tr}(\underline{b})=2lK_r$$

The sensitivity function for T_r could be derived by the partial differentiation of T_r with respect to the design variables. Also, the exact expression of the sensitivity function for the sub-functions have been summarized in Table 2.

$$\begin{aligned} \frac{\partial T_r}{\partial \underline{b}} &= \frac{\partial}{\partial \underline{b}}\left[\frac{g_{Tr}}{f_{Tr}}\right] \\ &= \frac{1}{f_{Tr}^2}\left[\frac{\partial g_{Tr}}{\partial \underline{b}}f_{Tr}-g_{Tr}\frac{\partial f_{Tr}}{\partial \underline{b}}\right] \end{aligned} \quad (32)$$

Table 2 Sensitivity Functions for g_{Tr} and f_{Tr} in T_r

Functions / Design Variables	g_{Tr}	f_{Tr}
m	l_fV	0
I	0	0
K_f	0	0
K_r	0	$2(l_f+l_r)$
l_f	mV	$2K_r$
l_r	0	$2K_r$

As the second process of the derivation of the sensitivity functions in the frequency domain, sensitivity of the phase must be considered. The sensitivity function for phase function could be obtained by partial differentiation of this function with respect to the design variables.

$$\frac{\partial}{\partial \underline{b}}\angle G_\delta^r=\left[\frac{P_r^2}{P_r^2+Q_r^2}\right]\frac{\partial}{\partial \underline{b}}\left(\frac{Q_r}{P_r}\right)-\left[\frac{R_r^2}{R_r^2+S_r^2}\right]\frac{\partial}{\partial \underline{b}}\left(\frac{S_r}{R_r}\right) \quad (33)$$

where,

$$\frac{\partial}{\partial \underline{b}}\left(\frac{Q_r}{P_r}\right)=\frac{1}{P_r^2}\left[\frac{\partial Q_r}{\partial \underline{b}}\cdot P_r-Q_r\cdot\frac{\partial P_r}{\partial \underline{b}}\right] \quad (34)$$

$$\frac{\partial}{\partial \underline{b}}\left(\frac{S_r}{R_r}\right)=\frac{1}{R_r^2}\left[\frac{\partial S_r}{\partial \underline{b}}\cdot R_r-S_r\cdot\frac{\partial R_r}{\partial \underline{b}}\right] \quad (35)$$

The partial derivatives of P_r, Q_r, R_r, and S_r were derived from Eq. (30). As a result, Eqs. (23) ~ (35) are used for all sensitivity analyses in the frequency domain.

4. Results of the Sensitivity Analysis

Various sensitivity analyses were carried about a nominal point for the front wheel vehicle system in the frequency domain. Vehicle data used in this research are listed in Table 3. These data are general for the conventional mid-size passenger car. From these data, we could see that the car shows an understeer characteristic and directional

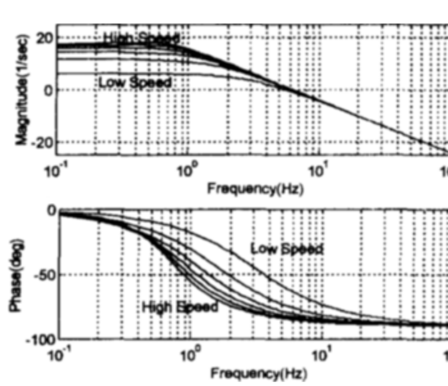

Fig. 2 Magnitude and phase change of the yaw rate w.r.t. vehicle speed (km/h)

stability for steering maneuver.

Table 3 Vehicle data list used for analyses

Design Variables	Data	Dimension
m	1,300	kg
I	2,100	$kg \cdot m^2$
K_f	40,000	N/rad
K_r	30,000	N/rad
l_f	1.01	m
l_r	1.65	m

Sensitivity information is evaluated in various vehicle speed ranges (20km/h~120km/h) with respect to the design variables. At first, the effects of each design variables for the yaw rate are examined. Secondly, a dominant design variable is checked by some comparisons of the results. Before the simulation of the sensitivity analysis, conventional frequency analyses are performed for magnitude and phase of the yaw rate with respect to the various vehicle are performed for magnitude and phase of the yaw rate with respect to the various vehicle speed. The results of magnitude and phase of the yaw rate are represented in Fig. 2. From these results, we could see that the magnitude and phase of the yaw rate are regularly changed with vehicle speed variation.

In case of yaw rate, the magnitude is positively increased as the vehicle speed increased, Also, we noticed that the magnitude for yaw rate has the dominatnt characteristic at about 1 Hz. In case of the phase for yaw rate, the phase lag is increased as the vehicle speed increased at about 1 Hz region. From this result and Eq. (14), we could see that the stability of the yaw rate response is always guaranteed. It means that the response of the yaw rate has minimum phase characteristics (Ogata, 1990, Slotion and Li, 1991, Frankil et al., 1994). After 10 Hz, the phases for all speed range are converged to-90 degree. From the trends of magnitude and phase of the yaw rate, we can conclude that the frequency reponess of the yaw rate are dominant at below the 1 Hz region.

4.1 The effects of the vehicle mass

Figure 3 shows the result of the yaw rate sensitivity with respect to the change of vehicle mass.

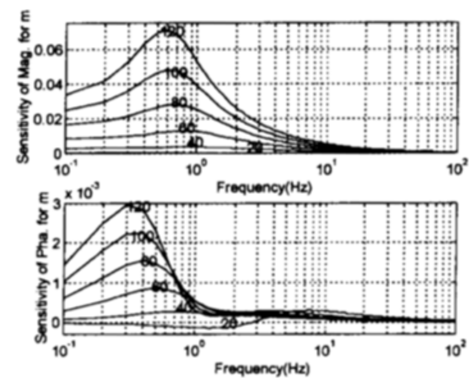

Fig. 3 Yaw rate sensitivity for m w.r.t. the change of vehicle speed (km/h)

The upper figure represents the magnitude sentivity of the yaw rate with respect to the vehicle mass is positively increased as the vehicle speed increased. An interesting point from this result is that the sensitivity value of magnitude for vehicle mass shows a maximum sensitivity value at about 1~2 Hz in km/h vehicle speed. Above 20 km/h speed, maximum sensitivity values for each speed are detected at the lower frequency range compared to the case of 20 km/h. But, all values are converged to the zero value as the input frequency increased. The lower giguer shows the result of the phase sensitivity with respect to the vehicle mass according to the speed change. In this result, the value at the low speed 20 km/h shows a negative value. At this speed, a zero sensitivity value is detected at about 3 Hz region. Above the 20 km/h speed, the sensitivity values are positively increased as speed increased. The peak sensitivity values occurred below the 1 Hz for all speed range. Like the case of sensitivity for magnitude, the values converged at zero as frequency increased.

4.2 The effects of the vehicle inertia

Figure 4 the result of the yaw rate sensitivity with respect to the change of vehicle inertia. In the upper figure, the sensitivity values are negatively increased as speed increased. From this result, we know that the sensitivity value of magnitude with respect to the vehicle inertia is dominant in the range of 1~4 Hz region. In the lower figure, the sensitivity of the phase is represented. The values are negarively increased as the vehicle speed increased. And the frequencies for the maximum sensitivity value are shifted downward as speed increased. All values for high frequency range are converged to the zero value. Like the magnitude case, dominant range exist in 1~4 Hz region.

4.3 The effects of the cornering stiffness of the front wheel

Figure 5 represents the yaw rate sensitivity with respect to the change of the cornering stiffiness of the front wheel. The sensitivity of the magnitude and phase are the upper part and the lower part of the Fig. 5, respectively. The sensitivity values for magnitude are positively increased as the vehicle speed increased. Any zero points of sensitivity value do not exist in this case. As the vehicle speed increased, the maximum sensitivity point is located at below 1 Hz. In case of the sensitivity for the phase, zero points of the sensitivity value for the 60~70 km/h speed range are detected at about 1~1.5 Hz region. Above 1 Hz, the sensitivity values for phase are decreased as the vehicle speed increased. As frequency continuously increased, all sensitivity values converged to the zero value. It must be noted that the response of the sensitivity for phase might be critical in the frequency range of 1~1.5 Hz region.

4.4 The effects of the cornering stiffness of the rear wheel

Figure 6 is the result of the yaw rate sensitivity with respect to the change of the cornering stiffiness of the rear wheel. The sensitivity values of yaw rate for cornering stiffness of the front wheel are positively increased as the vehicle speed increased. Any zero points of sensitivity value do not exits in this case. Above the 60 km/h, the maximum values occurred at below 1 Hz. In the lower figure, the sensitivity values of the phase are suggested. The values are positively increased as the vehicle speed increased. The frquency ranges for the maximum sensitivity values are continously decreased as the vehicles speed increased. Like the previous case, all maximum sensitivity values occurred at about 0.5~2 Hz region. In this case any zero points of the sensitivity value do not

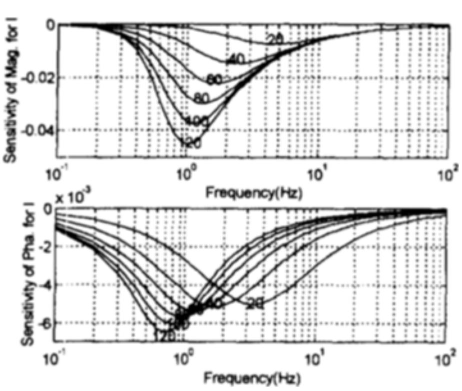

Fig. 4 Yaw rate sensitivity for I w.r.t. the change of vehicle speed (km/h)

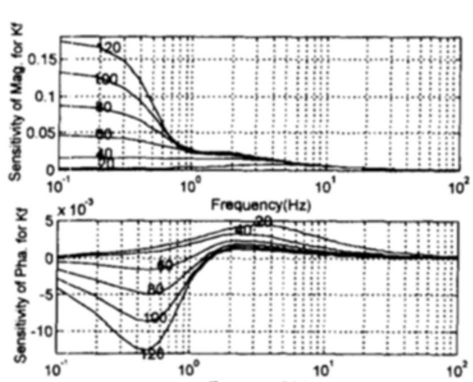

Fig. 5 Yaw rate sensitivity for K_f w.r.t. the change of vehicle speed (km/h)

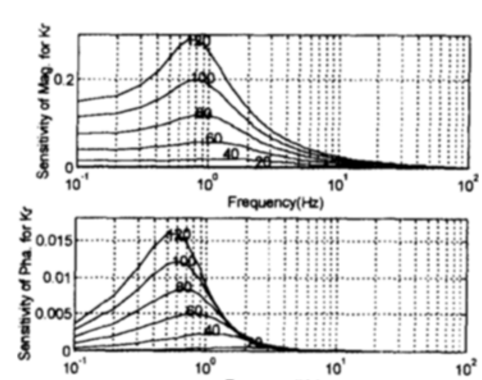

Fig. 6 Yaw rate sensitivity for K_r w.r.t. the change of vehicle speed (km/h)

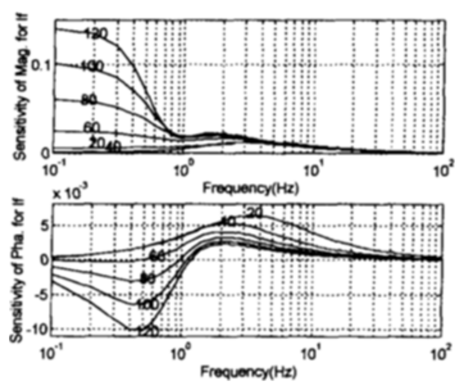

Fig. 7 Yaw rate sensitivity for l_f w.r.t. the change of vehicle speed (km/h)

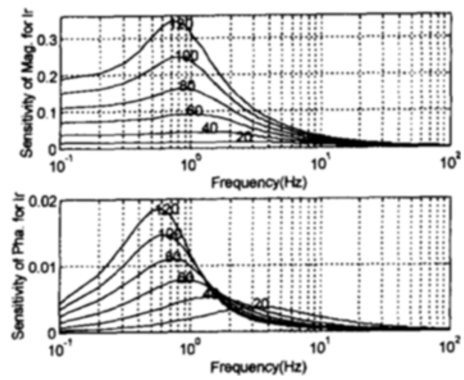

Fig. 8 Yaw rate sensitivity for l_r w.r.t. the change of vehicle speed (km/h)

exist.

4.5 The effects of the distance from c.g. to front wheel center

Figure 7 is the of the yaw rate sensitivity with respect to the change of the distance from center of gravity to front wheel center. In this case, the values positively increase with vehicle speed-up. Any zero points of sensitivity value do not exist in this case. The sensitivity values of the phase are shown in the lower gigure. The values positively decrease in low speed range (0~40 km/h) and negatively increase in high speed range (60~120 km/h). Like the previous case, all maximum sensitivity values occurred at about 0.5~3 Hz region. The zero points for sensitivity value are detected at about the 0.5~1.2 Hz region.

4.6 The effects of the distance from c.g. to rear wheel center

The result of the yaw rate sensitivity with respect to the change of the distance from center of gravity to rear wheel center is shown in Fig. 8. In this case, the values are positively increased with vehicle speed-up. Any zero points of sensitivity value are not detected. Particularly, in toe 20 km/h vehicle speed, the maximum value occurred at about 4 Hz. Above the 20 km/h, the values are increased as speed increased. But, the frequency ranges for the maximum sensitivity values are continuously decreased as the vehicle speed encreased. In the lower figure, the sensitivity values

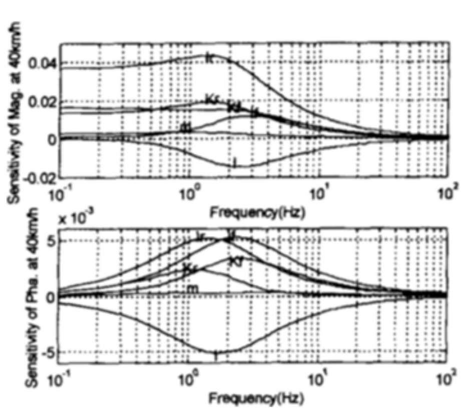

Fig. 9 Yaw rate sensitivity with respect to each design variables at 40km/h

of the phase are suggested. The values positively increased as the vehicle speed increased. like the previous case, all maximum sensitivity values occurred at about the 0.6~3 Hz region. In this case, any zero points of the sensitivity value do not exit.

4.7 Dominant design variable study

The dominant design variables could be examined beased on the sensitivity analyses for yaw rate of the vehicle center of gravity. In order to check the dominant design variable, a normalization process is required for the dimension matching. In kthis research. 1% perturbation of the design variables is performed for this process. The rank of the design variables occurred differently as vehicle speed increased. In this study,

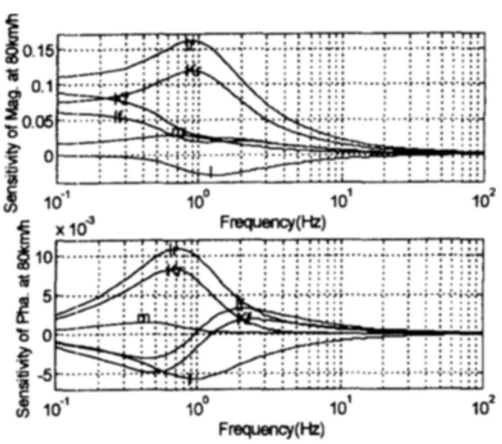

Fig. 10 Yaw rate sensitivity with respect to each design variables at 80km/h

only two cases of vehicle speed (40km/h and 80 km/h) are investigated for brevity.

In case of 40 km/h vehicle speed, the results are grouped in Fig. 9. From the upper figure, we could see that a dominant design variable for magnitude of the yaw rate is l_r. Next, I, l_f, K_r, K_f, and m are followed in turn. The sensitivities show quite different values at about the 1~2 Hz region. But as the frequency increased, the values are converged to zero. Any zero points were not detected in this vehicle speel. From the lower figure, we could search that a dominant design variable for phase of the yaw rate is l_r. Next, I, l_f, K_f, K_r, and m are followed consecutively. In this case, the sensitivities show most different values at about 1~3 Hz region.

In case of 80 km/h vehicle speed, the results are shown in Fig. 10. Like the previous case, we could se that a dominant design variable for magnitued is l_r. Next, K_r, I, K_f, l_f, and m are followed in turn. The sensitivities show quite different values at below 1 Hz. But as the frequency increased, the values are converged at zero. Any zero points of the sensitivity do not exist in this case. From the lower fegure, we could search that a dominant design variable for phase of the yaw rate is l_r. Next, K_r, I, K_f, l_f, and m are followed consecutively. In this case, the sensitivities have most different values at below the 1 Hz. Also, although K_f, and l_f have minor rank for the phase sensitivity, they must be carefully treated in analysis and control stage because it has a zero sensitivity value at about the 1~1.2 Hz region. But, the values of the high frequency are converged to zero like the magnitude case.

Form Figs. 9 and 10, we migh conclude that the sensitivity values for design variables are changed differently as the vehicle speed up and down. As an example, the vehicle inertia effect is reduced as the vehicle speed incresed. This changing point of the vehicle response is detected in this study at about 1 Hz. Also, we could note that dominant frequency range is changes from above 1 Hz to below 1 Hz as the vehicle speed increased. That is to say, the dominant frequency range go downward as the vehicle speed increased. So, a designer must consider the sensitivity of vehicles with various vehicle speed during design stage. These results could be used as a basis for the design of the vehicle which has more enhanced frequency response for handling performances and the new-robust control logic for the external disturbances. Also, this may be used by vehicle designer for re-design and new-design of the vehicle.

5. Conclusions

In this study, the sensitivity analysis for the yaw rate of the front wheel steering vehicle system are carride out in the frequency domain. From the sensitivity analysis, useful sensitivity information is obtained at various vehicle speeds with respect to the design variables. First, the effects of each design variables on state variables are examined. Secondly, a dominant design variable is checked by some comparisons of the results. In the results, zero points of the state sensitivity variable which may be a critical point for vehicle design and control are detected. These points must be carefully considered as a design base. Also, we could see that the effects of the distance form center of gravity to rear wheel center and from center of gravity to front wheel center to the state sensitivity are the most sensitive design variables. The vehicle inertia has the second most sensitivity in a klow speed range. But its importance is continuously reduced as the vehicle speed increased. In case of controlling the vehicle, the zero points of the state sensitivity results must be outside from the nominal driving conditions because the sever problems may occure at these points. In case of designing the vehicle, a modification of the vehicle wheel base must be carefully treated for the reasonable vehicle design. Also, we could see that the frequency response of the yaw rate is always stable. It means that the response has minimum phase characteristics owing to the stable zero dynamics. And, we could see that the dominant frequency range become lower as the speed increased. So, a vehicle must be considered with various vehicle speeds separately. This research may be used for the optimization part in the frequency domain and for the control part of the vehicle.

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